Published by Institute of Physics Publishing for SISSA

Received: *February 14, 2005*
Accepted: *March 29, 2005*
Published: *May 4, 2005*

Brane dynamics, central charges and E_{11}

Peter West

Department of Mathematics
King's College, London WC2R 2LS, U.K.
E-mail: `pwest@mth.kcl.ac.uk`

Abstract: We consider a theory in which supersymmetry is partially spontaneously broken and show that the dynamical fields in the same supersymmetric multiplet as the Goldstino are Goldstone bosons whose corresponding generators are central charges in the underlying supersymmetry algebra. We illustrate how this works for four dimensional Born-Infeld theory and five brane of M theory. We conjecture, with supporting arguments, that the dynamics of the branes of M theory can be extended so as to possess an E_{11} symmetry.

Keywords: M-Theory, p-branes, Supersymmetry and Duality.

JHEP03(2005)077

Contents

1. Introduction

Given a theory with a symmetry algebra G which is spontaneously broken to the sub-algebra H then it is usually the case that the low energy theory is a non-linear realisation of G with local sub-algebra H [1]. Hence, if one is studying a phenomenon which is of this type then, by examining the low energy behaviour, one can discover the symmetries of the theory without having to understand the underlying dynamics. Indeed, it was using this idea that some of the most important symmetries of particle physics were first established.

One of the very first papers on supersymmetry constructed the non-linear realisation which results when a four dimensional $N = 1$ supersymmetry theory spontaneously breaks all its of supersymmetry [7]. Superbranes can be thought of as a defects in superspace in which supersymmetry is partially spontaneously broken and as such should be described by a non-linear realisation. A selection of the substantial number of papers on this viewpoint, when half the supersymmetry is spontaneously broken, are given in references [1 − 6, 22]. Following these it has become well known that the fermion field in superbrane dynamics is the Goldstino for broken supersymmetry and the transverse scalar fields are the Goldstone boson for broken translations. In more recent years branes that include world volume gauge fields have played an important role. The most important examples are the D-branes in ten dimensions and the five brane of M theory. Such branes can be also described as a non-linear realisation and it was shown [8] that the derivatives of the world volume gauge fields and also the usual transverse scalars fields, are Goldstone bosons for automorphism of the supersymmetry algebra. However, this paper did not find a symmetry corresponding to the world-volume gauge fields themselves.

In section 2 we will consider a theory in which supersymmetry is spontaneously broken and present a simple argument that shows that the dynamical bosonic fields in the same multiplet as the Goldstino are Goldstone bosons whose corresponding generators are the central charges in the underlying supersymmetry algebra. This argument applies to the world volume gauge fields of superbranes and we will show in section 3 how this occurs in the general non-linearly realised formalism and in particular in section 4 for the five brane of M theory and the four dimensional Born-Infeld theory. Thus one finds that all the dynamical bosonic fields of branes have a common origin in that they are the Goldstone bosons corresponding to the central charges.

It has been proposed that M theory possesses an underlying E_{11} symmetry; the low level fields in this non-linear realisation being precisely those of the maximal supergravity theories [9]. The latter are just the low energy description of the closed string theory, however, it has been known since early on in the development of string theory that open string scattering leads to closed strings and so one might suspect that open strings, when suitably extended, should also possess an E_{11} symmetries. In particular, one might hope that this would show up in the effective action for open superstrings, namely the Born-Infeld actions, or more generally in the dynamics of the branes of M theory.

There is considerable evidence that all the usual brane charges are contained in a single fundamental representation of E_{11} denoted l_1 [11, 14, 16]. This multiplet contains at its lowest levels the space-time translations, and the two and five form central charges of the eleven dimensional supersymmetry algebra and then an infinite number of more exotic objects. Regarding branes as a result of spontaneously symmetry breaking it is a natural generalisation to think of the dynamics of their bosonic fields as constructed from Goldstone bosons corresponding to generators which belong to the l_1 representation. The result mentioned above for the five brane is an encouraging sign in this direction as one finds that its dynamics contains the transverse scalars and the world volume gauge field both of which occur as the first two fields in the l_1 representation and their interpretation as Goldstone bosons for the central charges implies that they occur in the non-linear realisation in a way that is a low level truncation of an E_{11} formulation.

However, at first sight, the situation for the two brane of M theory appears less promising. However, in section 5 we sketch the low level E_{11} non-linear realisation based on the l_1 representation appropriate to the two brane and recover the usual dynamics for the bosonic fields. Finally, in section 6 we discuss some of the implications of the conjecture advanced above.

2. Central charges and Goldstone fields

Let us consider a supersymmetric theory in which some, but not all, of the supersymmetry is spontaneously broken. We denote the preserved supersymmetry generators by Q_α and the broken supersymmetry generators by $Q_{\alpha'}$. The indices α and α' may contain internal as spinor indices. The Goldstino, that is the field corresponding to the broken supersymmetry is denoted by $\bar\lambda^{\alpha'}$ and let $A^\bullet$ be one of the dynamical bosonic fields in the same multiplet as the Goldstino under the preserved supersymmetry Q_α. Here $\bullet$ labels any indices on the bosonic field. On dimensional and Lorentz symmetry grounds it follows that the transformation of $A^\bullet$ under a preserved supersymmetry with parameter $\bar\epsilon^\alpha$ is of the form

$$\delta A^\bullet = \bar\epsilon^\alpha (\gamma^\bullet C^{-1})_{\alpha\beta'} \bar\lambda^{\beta'} \, . \tag{2.1}$$

Now by definition the Goldstino transforms under the broken supersymmetry as $\delta\bar\lambda^{\alpha'} = \eta^{\alpha'} + \cdots$ where $+\cdots$ denotes field dependent terms. Consequently, the commutator of a broken and an unbroken supersymmetry on $A^\bullet$ takes the form

$$[\delta_\epsilon, \delta_\eta] A^\bullet = -\bar\epsilon^\alpha (\gamma^\bullet C^{-1})_{\alpha\beta'} \bar\eta^{\beta'} + \cdots \, . \tag{2.2}$$

This implies that there is a central charge $Z_\bullet$ in the underlying supersymmetry algebra which occurs as

$$\{Q_\alpha, Q_{\beta'}\} = Z_\bullet (\gamma^\bullet C^{-1})_{\alpha\beta'} \, . \tag{2.3}$$

The action of $Z_\bullet$ on $A^\bullet$ is to shift it by a constant and consequently we can interpreted $A^\bullet$ as the Goldstone field corresponding to the generator $Z_\bullet$. Thus we conclude that a dynamical field in the same supersymmetry multiplet as the Goldstino must be a Goldstone boson whose corresponding generator is one of the central charges that occurs in the anti-commutator of the preserved and broken supersymmetries.

We can also identify the derivatives of the dynamical Goldstone fields. Let us assume that there exists a symmetry of the theory that rotates the Goldstino into the superspace coordinate θ^α, namely

$$\delta\bar\lambda^{\alpha'} = \theta^\beta L_\beta{}^{\alpha'} \, . \tag{2.4}$$

Carrying out the commutator of a supersymmetry transformation and the above rotation we find that

$$[\delta_\epsilon, \delta_L] \bar\lambda^{\alpha'} = \bar\epsilon^\beta L_\beta{}^{\alpha'} + \cdots \, . \tag{2.5}$$

If we denote the generator of the rotation by $R_{\alpha'}{}^\beta$ we must conclude that it occurs in the underlying algebra as

$$[Q_\gamma, R_{\alpha'}{}^\beta] = \delta_\gamma^\beta Q_{\alpha'} \, . \tag{2.6}$$

Hence it is one of the automorphism of the supersymmetry algebra studied in [8].

The Goldstino $\bar\lambda^{\alpha'}$ and Goldstone boson $A^\bullet$ belong to a supermultiplet whose first component is $A^\bullet$. Denoting the superfield and its first component by the same symbol equation (2.1) implies that $D_\alpha A^\bullet = (\gamma^\bullet C^{-1})_{\alpha\beta'} \bar\lambda^{\beta'}$. As a consequence of equation (2.4), the spinorial derivative of the Goldstino transforms as

$$\delta_L D_\alpha \lambda^{\beta'} = L_\alpha{}^{\beta'} + \cdots \, . \tag{2.7}$$

As such, the spinorial derivative of the Goldstino is the Goldstone boson for the automorphism of the supersymmetry algebra with generator $R_{\alpha'}{}^\beta$. However, the spinorial derivative, or equivalently the supersymmetry variation, of the Goldstino contain the space-time derivatives of the dynamical Goldstone fields and any auxiliary fields that my be present. As a consequence, we conclude that the latter fields which do not occur with derivatives in the supersymmetry variation, are also Goldstone fields of some of the automorphism of the supersymmetry algebra that mixes the preserved and broken supercharges.

Let us also consider a rotation of the form

$$\delta_{\hat{L}} A^{\bullet} = \hat{L}^{\bullet}{}_{n} X^{n} \tag{2.8}$$

where X^n is the space-time coordinate of the theory and let $R_{\bullet}{}^{n}$ be the associated generator. The commutator of a space-time translation with parameter ζ^n and the above rotation is

$$[\delta_{\zeta}, \delta_{\hat{L}}] A^{\bullet} = \hat{L}^{\bullet}{}_{n} \zeta^{n} + \cdots . \tag{2.9}$$

This is a shift in $A^{\bullet}$ that is generated by the previously identified generator $Z_{\bullet}$ and so we find that

$$[P_n, R_{\bullet}{}^{m}] = \delta_n^m Z_{\bullet} \tag{2.10}$$

It follows that $\partial_n A^{\bullet}$ transforms as $\delta_{\hat{L}} \partial_n A^{\bullet} = \hat{L}^{\bullet}{}_{n} + \cdots$ and so can be identified as the Goldstone boson whose generator is $R_{\bullet}{}^{n}$.

This last result is consistent with our earlier considerations, since the super-Jacobi identity involving Q_α, Q_β and $R_{\gamma'}{}^{\beta}$ implies using equation (2.6) that $R_{\gamma'}{}^{\beta}$ rotates $Z_{\alpha\beta} \equiv \{Q_\alpha, Q_\beta\}$ into $Z_{\alpha\gamma'} \equiv \{Q_\alpha, Q_{\gamma'}\}$. Since $Z_{\alpha\beta}$ include the translation generator P_n certain of the $R_{\alpha'}{}^{\beta}$ will rotate P_n into the central charges $Z_{\alpha\beta'}$ as in equation (2.10). Expanding $R_{\alpha'}{}^{\beta}$ in terms of Clifford algebra elements we find a set of generators with vector indices which are totally anti-symmetrised and as a result only the totally anti-symmetric part of $R_{\bullet}{}^{m}$ can be identified with part of $R_{\alpha'}{}^{\beta}$. In a given model only a sub-set of the allowed possible central charges may be present and it can happen that some parts of $R_{\alpha'}{}^{\beta}$ do not induce any transformations on P_a. As such, the Goldstone fields corresponding to these generators are not space-time derivatives of the central charge Goldstone fields even though, as explained above, they are the Goldstone bosons for $D_\alpha \bar{\lambda}^{\beta'}$. The obvious interpretation of these Goldstone fields is that they correspond to auxiliary fields.

In this section, we have not explicitly used the machinery of non-linear realisations, however, in the next section we will give the theory of non-linear realisations applied to the current context and we will see how the results found in this section emerge from the general formalism.

This interpretation of auxiliary fields as part of the automorphism algebra may provide a new way of finding auxiliary fields.

3. The general formalism

We first recall the description of branes as non-linear realisations as given in reference [8]. We consider the supersymmetry algebra, which obeys the relations

$$\{Q_{\underline{\alpha}}, Q_{\underline{\beta}}\} = Z_{\underline{\alpha\beta}}\,, \qquad [Q_{\underline{\gamma}}, Z_{\underline{\alpha\beta}}] = 0\,, \qquad [Z_{\underline{\alpha\beta}}, Z_{\underline{\gamma\delta}}] = 0\,, \tag{3.1}$$

and denoted it by $\underline{K}$, We also consider the automorphism algebra $\underline{H}$ of $\underline{K}$ which obeys the relations

$$[Q_{\underline{\alpha}}, R_{\underline{\gamma}}{}^{\underline{\delta}}] = \delta_{\underline{\alpha}}{}^{\underline{\delta}} Q_{\underline{\gamma}}\,, \qquad [Z_{\underline{\alpha\beta}}, R_{\underline{\gamma}}{}^{\underline{\delta}}] = \delta_{\underline{\alpha}}{}^{\underline{\delta}} Z_{\underline{\gamma\beta}} + \delta_{\underline{\beta}}{}^{\underline{\delta}} Z_{\underline{\alpha\gamma}}\,. \tag{3.2}$$

The generators of $\underline{K}$ and $\underline{H}$ together form the algebra $\underline{G}$ from which the non-linear realisation is constructed. The central charge generators $Z_{\underline{\alpha\beta}}$ include the space-time momentum generators $P_{\underline{a}}$. It is easy to verify that such an algebra obeys the generalized super Jacobi identities.

Expanding $Z_{\underline{\alpha\beta}}$ out in terms of the enveloping algebra of the relevant Clifford algebra we find that it contains a set of generators which are totally anti-symmetric in their vector indices:

$$Z_{\underline{\gamma}}{}^{\underline{\delta}} = \sum_p \sum_{\underline{n}_1\cdots\underline{n}_p} (\gamma^{\underline{n}_1\cdots\underline{n}_p} C^{-1})_{\underline{\gamma}}{}^{\underline{\delta}} Z_{\underline{n}_1\cdots\underline{n}_p} \,. \tag{3.3}$$

Similarly, we may expand $R_{\underline{\gamma}}{}^{\underline{\delta}}$, namely

$$R_{\underline{\gamma}}{}^{\underline{\delta}} = \sum_p \sum_{\underline{n}_1\cdots\underline{n}_p} (\gamma^{\underline{n}_1\cdots\underline{n}_p})_{\underline{\gamma}}{}^{\underline{\delta}} R_{\underline{n}_1\cdots\underline{n}_p} \,. \tag{3.4}$$

If all possible central charges are allowed the generators $Z_{\underline{\alpha\beta}}$ form the most general symmetric matrix and the automorphism group is $GL(c_d)$ where c_d is the number of supercharges. However, it is often the case that we require only a sub-set of all the possible central charges in which case the automorphism group is reduced. Indeed, if the only central charge is the momentum then the right-hand side of the anti-commutator of two supercharges is $\gamma^{\underline{a}} P_{\underline{a}}$, then the most general automorphism group is by definition the spin group. This natural enlargement of the Lorentz algebra when more central charges are present acts as a brane rotating symmetry [8, 15].

Let us consider the action of one of the automorphisms $R^\bullet$ that occurs in equation (3.4) where $\bullet$ encodes the indices. Equation (3.2) then takes the form

$$[Z_{\underline{\alpha\beta}}, R^\bullet] = (\gamma^\bullet)_{\underline{\alpha}}{}^{\underline{\delta}} Z_{\underline{\delta\beta}} + (\gamma^\bullet)_{\underline{\beta}}{}^{\underline{\delta}} Z_{\underline{\alpha\delta}} \,. \tag{3.5}$$

We may rewrite this equation as

$$[Z_{\underline{\alpha\beta}}, R^\bullet] = (\gamma^\bullet Z C - \eta\epsilon Z C \gamma^\bullet)_{\underline{\alpha}}{}^{\underline{\delta}} (C^{-1})_{\underline{\delta\beta}} \tag{3.6}$$

where the charge conjugation matrix C satisfies $C^T = -\epsilon C$ and $(\gamma^\bullet C^{-1})^T = \eta\gamma^\bullet C^{-1}$. Given a set of central charges $Z_{\underline{\alpha\beta}}$ we may use the last equation to find what is the maximal automorphism algebra. We note that it depends on the transposition properties of $\gamma^\bullet$ and C both of which are dimension dependent.

We divide the generators in $\underline{K}$ into $Q_{\underline{\alpha}} = (Q_\alpha, Q_{\alpha'})$ and $Z_{\underline{\alpha\beta}} = (Z_{\alpha\beta}, Z_{\alpha\beta'}, Z_{\alpha'\beta'})$ and the generators of $\underline{H}$ as $R_{\underline{\gamma}}{}^{\underline{\beta}} = (R_\gamma{}^\beta, R_{\gamma'}{}^\beta, R_\gamma{}^{\beta'}, R_{\gamma'}{}^{\beta'})$. In this decomposition Q_α and $Q_{\alpha'}$ are the preserved and broken supersymmetry generators respectively. The decomposition of the spinor indices corresponding to the breaking of the underlying spin algebra. The generators Q_α, $Z_{\alpha\beta}$ and a suitable sub-algebra H of $\underline{H}$ are preserved and we denote the algebra of these generators by G.

We consider the non-linear realisation of n $\underline{G}$ with local sub-algebra H and so consider the group element

$$\begin{aligned} g &= e^{X^{\alpha\beta} Z_{\alpha\beta} + \theta^\alpha Q_\alpha} e^{X^{\alpha\beta'} Z_{\alpha\beta'} + X^{\alpha'\beta'} Z_{\alpha'\beta'} + \Theta^{\alpha'} Q_{\alpha'}} e^{\phi\cdot R} \\ &\equiv g_p e^{X^{\alpha\beta'} Z_{\alpha\beta'} + X^{\alpha'\beta'} Z_{\alpha'\beta'} + \Theta^{\alpha'} Q_{\alpha'}} e^{\phi\cdot R} \end{aligned} \tag{3.7}$$

where $\phi\cdot R = \phi_{\underline{\delta}}{}^{\underline{\gamma}} R_{\underline{\gamma}}{}^{\underline{\delta}}$ is a sum that includes all the generators in $\underline{H}$, except for those in H. Although the group element contains $Z_{\alpha\beta}$ and Q_α these do correspond to preserved symmetries and their role is to introduce superspace into the theory. In particular , $X^{\alpha\beta'}$, $X^{\alpha'\beta'}$, $\Theta^{\alpha'}$ and ϕ depend on this superspace.

JHEP03(2005)077

To illustrate the results of the previous section, it will suffice to consider the linearised approximation in which we keep terms only to first order in the dynamical fields. Hence, $X^{\alpha\beta}$ and θ^α are of order zero and all other fields are of order one. To this order the Cartan forms are given by

$$\begin{aligned} g^{-1}dg = &\, dz^\pi (E_\pi{}^a P_a + E_\pi{}^\alpha Q_\alpha) + E^N \nabla_N \Theta^{\alpha'} Q_{\alpha'} + \\ &+ E^N \nabla_N X^{\alpha\beta'} Z_{\alpha\beta'} + E^N \nabla_N X^{\alpha'\beta'} Z_{\alpha'\beta'} + E^N \nabla_N \phi \cdot R \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} g_p^{-1} dg_p &\equiv dz^\pi (E_\pi{}^a P_a + E_\pi{}^\alpha Q_\alpha) \equiv E^a P_a + E^\alpha Q_\alpha \\ &= d\theta^\alpha Q_\alpha + \left(dX^a - \frac{1}{2} d\theta^\alpha (\gamma^a C^{-1}\theta)_\alpha \right) P_a \,, \end{aligned} \tag{3.9}$$

and

$$\nabla_\gamma X^{\alpha\beta'} = D_\gamma X^{\alpha\beta'} - \delta_\gamma^\alpha \Theta^{\beta'} \tag{3.10}$$

$$\nabla_\alpha \Theta^{\beta'} = D_\alpha \Theta^{\beta'} + \phi_\alpha{}^{\beta'} \tag{3.11}$$

$$\nabla_{\gamma\delta} X^{\alpha\beta'} = \partial_{\gamma\delta} X^{\alpha\beta'} + \frac{1}{2}(\delta_\gamma^\alpha \phi_\delta{}^{\beta'} + \delta_\delta^\alpha \phi_\gamma{}^{\beta'}) \,. \tag{3.12}$$

We also find that $\nabla_{\gamma\delta}\Theta^{\alpha'} = \partial_{\gamma\delta}\Theta^{\alpha'}$ and $\nabla_N \phi_{\underline{\alpha}}{}^{\underline{\beta}} = D_N \phi_{\underline{\alpha}}{}^{\underline{\beta}}$, although these two quantities will play little further role in what follows. In the above we have denoted the coordinates of superspace by $z^\pi = (X^a, \theta^\alpha)$, the index range of $N = (a, \alpha)$ and the world indices, such as π, by the same range since we are working with the linearised approximation. In deriving these results we have defined the covariant derivative in superspace by

$$df = dz^\pi \partial_\pi f = E^N D_N f \tag{3.13}$$

for any function f of superspace and where $E^N = dz^\pi E_\pi{}^N$. It is a consequence of the linearized analysis that $\phi_\gamma{}^{\beta'}$ is the only part of $\phi_{\underline{\gamma}}{}^{\underline{\beta}}$ that enters any of these expressions.

The dynamics is given by setting

$$\nabla_\gamma X^{\alpha\beta'} = 0 = \nabla_\alpha \Theta^{\beta'} \qquad \text{or equivalently} \qquad D_\gamma X^{\alpha\beta'} = \delta_\gamma^\alpha \Theta^{\beta'} \,, \qquad D_\alpha \Theta^{\beta'} = -\phi_\alpha{}^{\beta'} \,. \tag{3.14}$$

The Cartan forms transform under H and given our choice of automorphism algebra the above set of constraints is invariant.

The constraints of equation (3.14) make contact with the general discussion of the previous section. The first of these equations implies that the supersymmetry variation of $X^{\alpha\beta'}$, denoted $A^\bullet$ in the previous section, has the form of equation (2.1). This equation also is often sufficient to determine the dynamics of the brane. The second of these equations tells us that the spinorial derivative of the Goldstino is part of the automorphism algebra and so belongs to $\underline{H}/H$. By taking the spinorial covariant derivatives of the first equation we find an alternative expression for the spinoral covariant derivative of the Goldstino which involves the space-time derivatives of $X^{\alpha\beta'}$ and as such we conclude that the latter are part of the automorphism algebra [8]. This can be viewed as an example of the inverse

JHEP03(2005)077

Higgs mechanism [23]. Although in reference [8] a general form for $X^{\alpha\beta'}$ was allowed in the general formalism it was then assumed that these fields contained only the transverse fields $X^{a'}$ or, equivalently that the only active central charges were the space-time momenta. However, in this paper it was also suggested [8] that one should take a more general form for $Z_{\alpha\beta'}$, or $X^{\alpha\beta'}$, and it is this suggestion that is implemented here. In the examples given below we will find that for some theories these extra generators play an important role in that their corresponding fields are the world volume gauge fields that occur in some branes.

4. Examples

Certain aspects of the general scheme set out above have occurred in a number of papers in the literature. In particular, it is well known following the two- and four-dimensional examples worked out reference [2–6] that the transverse scalar fields that occur in brane actions are the Goldstone bosons for broken translations. It is obvious, but not always stressed that these broken translations must occur in the supersymmetry algebra of superbranes as central charges. A more recent example of this phenomenon is the construction of the $N = 1, D = 4$ supermembrane [5].

The non-linear realisation that leads to a chiral superfield in four dimensions, whose components we denote by $(\mathcal{A}; \chi; \mathcal{F})$, was worked out in [4, 3]. In these papers the authors considered a four dimensional $N = 2$ supersymmetry algebra with one complex scalar central charge Z which could be viewed as a six dimensional supersymmetry algebra, the complex central charge playing the role of the additional momenta. This supersymmetry algebra possessed the preserved generators $P_a, Q_A, Q_{\dot{A}}$ and J_{ab} and broken generators $S_A, Q_{\dot{A}}$, Z. Its automorphism algebra was SO(1,5)$\otimes$ SU(2) broken to SO(1,3)$\otimes$ SO(2)$\otimes$U(1). They found that the Goldstino is the fermion of the chiral multiplet $\chi_A, \chi_{\dot{A}}$, the Goldstone boson corresponding to the central charge is the complex scalar $\mathcal{A}$ and the Goldstone boson for the broken SU(2) part of the automorphism algebra are the complex auxiliary field $\mathcal{F}$. Furthermore, they noted that the remaining broken automorphisms were the space-time derivatives of the $\mathcal{A}$. This particular example possesses many features of the general scheme set about above, with the exception, like all previous work, that it involves only scalar central charges.

Below we will give three examples that illustrate the general scheme. We first give the M2 brane for comparison and then give the M five brane and finally the four dimensional Born-Infeld theory. Both the latter cases possess gauge fields which are the Goldstone bosons corresponding to central charges.

4.1 The M2 brane

We briefly recall how the M2 brane works. In this case the underlying algebra is

$$\{Q_{\underline{\alpha}}, Q_{\underline{\beta}}\} = (\gamma^{\underline{a}} C^{-1})_{\underline{\alpha}\underline{\beta}} P_{\underline{a}}\,. \tag{4.1}$$

Hence the only central charges are the translations and correspondingly we take $X^{\underline{\alpha}\underline{\beta}} = (C\gamma^{\underline{a}})^{\underline{\alpha}\underline{\beta}} X_{\underline{a}}$. The maximal automorphism algebra $\underline{H}$ is just the Lorentz algebra SO(1,10), or strictly speaking the spin algebra spin (1,10) for which $R_{\underline{\gamma}}{}^{\underline{\delta}} = \sum_{\underline{ab}} (\gamma^{\underline{ab}})_{\underline{\gamma}}{}^{\underline{\delta}} J_{\underline{ab}}$. Hence, the only non-zero $\phi_{\underline{\gamma}}{}^{\underline{\delta}}$ are the $\phi_{\alpha}{}^{\beta'} = \sum_{ab'} (\gamma^{ab'})_{\alpha}{}^{\beta'} \phi_{ab'}$. The preserved sub-algebra of spin(1,10) is $H =$ spin(1,2)$\otimes$ spin (8).

The constraints of equation (3.14) transform covariantly under spin(1,2)$\otimes$ spin (8) and so adopting these gives a set of equations which is invariant under the full non-linearly realised algebra. The first constraint implies that

$$D_\gamma X^{a'} = \frac{1}{16} (\gamma^{a'} C^{-1})_{\gamma\delta'} \Theta^{\delta'} , \tag{4.2}$$

which is indeed implies the correct equations of motion for the linearised dynamics. While the second equation implies that

$$D_\alpha \Theta^{\beta'} = -(\gamma^{ab'})_\alpha{}^{\beta'} \phi_{ab'} . \tag{4.3}$$

Applying a spinorial covariant derivative to equation (4.2) and using equation (4.3) we find that $\partial_m X^{n'} \sim -\phi_m{}^{n'}$ confirming that the space-time derivatives of $X^{a'}$ are the Goldstone boson $\phi_{ab'}$ corresponding to the automorphism $J_{ab'}$. In other words the space-time derivatives of $X^{a'}$, i.e. $\partial_b X^{a'}$ belong to the coset spin(1,10)/spin(1,2)$\otimes$spin(8). We also find that the constraints of equations (4.2) and (4.3) imply that $\nabla_m X^{a'} = 0$

4.2 The M5 brane

The underlying supersymmetry algebra $\underline{K}$ for the five brane is

$$\{Q_{\underline{\alpha}}, Q_{\underline{\beta}}\} = (\gamma^{\underline{a}} C^{-1})_{\underline{\alpha\beta}} P_{\underline{a}} + (\gamma^{\underline{ab}} C^{-1})_{\underline{\alpha\beta}} Z_{\underline{ab}} + (\gamma^{\underline{a}_1 \cdots \underline{a}_5} C^{-1})_{\underline{\alpha\beta}} Z_{\underline{a}_1 \cdots \underline{a}_5} \tag{4.4}$$

Although we consider $P_{\underline{a}}$ for $\underline{a} = 0, \ldots, 10$ we will take the other central charges to carry indices only over the range $\underline{a} = 0, \ldots, 5$. We must decompose the eleven dimensional Clifford algebra into one that keeps manifest the Clifford algebra appropriate to the five brane i.e spin(1,10) into spin(1,5)$\otimes$ spin(5). This results in a corresponding decomposition of the spinor index $\underline{\alpha} = (\alpha, \alpha')$ and then $\chi_\alpha \to \chi_{\alpha i}$ and $\chi_{\alpha'} \to \chi_i^\alpha$. For the fivebrane $\alpha = 1, \ldots, 4$ are the Weyl projected spinor indices of Spin(1,5) and $i = 1, \ldots, 4$ are the indices of the internal group Usp(4) = Spin(5).

Applying this decomposition to the supersymmetry algebra we find that the preserved supercharges $Q_{\alpha i}$ obey the anti-commutator

$$[Q_{\alpha i}, Q_{\beta j}] = \eta_{ij} (\gamma^a)_{\alpha\beta} P_a + \eta_{ij} (\gamma^{a_1 \cdots a_5}) Z_{a_1 \ldots a_5} = \eta_{ij} (\gamma^a)_{\alpha\beta} \hat{P}_a \tag{4.5}$$

where $\hat{P}_a = P_a - \epsilon_a{}^{b_1 \ldots b_5} Z_{b_1 \ldots b_5}$. Although we have a five form in the preserved supersymmetry algebra it can be absorbed to leave just a usual translation. However, the five form will reappear in the supersymmetry algebra for two broken supersymmetry generators.

$$[Q^\alpha{}_i, Q^\beta{}_j] = \eta_{ij} (\bar{\gamma}^a)^{\alpha\beta} (P_a + \epsilon_a{}^{b_1 \ldots b_5} Z_{b_1 \ldots b_5}) . \tag{4.6}$$

For the anti-commutator of a broken and an unbroken supersymmetry we find that

$$[Q_{\alpha i}, Q^\beta{}_j] = (\gamma^{a'})_{ij} \delta_\alpha^\beta P_{a'} + \eta_{ij} (\gamma^{a_1 a_2})_\alpha{}^\beta Z_{a_1 a_2} . \tag{4.7}$$

The general decomposition $X^{\underline{\alpha\beta}} = X^{\bullet}(C\gamma_{\bullet})^{\underline{\alpha\beta}}$ takes on a restricted form corresponding to the form of the anti-commutators in equation (4.5), (4.6) and (4.7). In particular in the world volume we take $X^{\alpha\beta} \equiv X^{\alpha i\beta j} = \hat{X}^a\eta^{ij}(\bar{\gamma}_a)^{\alpha\beta}$ and so we have in effect just the usual coordinates of space-time. Corresponding to equation (4.7), we take

$$X^{\alpha\beta'} \equiv X^{\alpha i}{}_{\beta}{}^{j} = -X^{a'}(\gamma_{a'})^{ij}\delta^{\alpha}_{\beta} + \eta^{ij}(\bar{\gamma}^{ab})^{\alpha}{}_{\beta}B_{ab}\,. \tag{4.8}$$

We will recognise $X^{a'}$ as the transverse scalars of the five brane and B_{ab} as its world volume gauge field.

Taking into account the restricted range of the indices on the central charges, the automorphism algebra of the supersymmetry algebra of equation (4.4) is generated by

$$R_{\underline{\alpha}}{}^{\underline{\beta}} = J_{\underline{ab}}(\gamma^{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}} + R_{a_1a_2a_3}(\gamma^{a_1a_2a_3})_{\underline{\alpha}}{}^{\underline{\beta}} \tag{4.9}$$

where $R_{a_1a_2a_3}$ is anti-self dual. In particular, we have $R_{\alpha'}{}^{\beta} = J_a{}^{b'}(\gamma_{b'})_i{}^j(\bar{\gamma}^a)^{\alpha\beta}R_{a_1a_2a_3} \times \delta_i^j(\bar{\gamma}^{a_1a_2a_3})^{\alpha\beta}$. Indeed, using equation (3.6) one can verify that once one introduces the two form central charge one must also include the five form central charge. The corresponding Goldstone fields required at lowest order have the form

$$\phi_{\alpha}{}^{\beta'} \equiv \phi_{\alpha i\beta}{}^{j} = -\phi_a{}^{b'}(\gamma_{b'})_i{}^j(\gamma^a)_{\alpha\beta} + \phi_{a_1a_2a_3}\delta_i^j(\gamma^{a_1a_2a_3})_{\alpha\beta} \tag{4.10}$$

where $\phi_{a_1a_2a_3}$ is self-dual.

Using equation (4.8), equation (3.10) implies that the equation

$$D_{\gamma k}X^{a'} = \frac{1}{16}(\gamma^{a'})_k{}^j\Theta_{j\gamma} \tag{4.11}$$

and also

$$D_{\gamma k}B_{ab} = \frac{1}{16}(\gamma_{ab})^{\beta}{}_{\gamma}\Theta_{k\beta}\,, \tag{4.12}$$

both of which are consistent with equation (2.1). Using equation (4.10), the condition of equation (3.11) then implies

$$D_{\alpha i}\Theta^{\beta}{}_{j} = -\phi_a{}^{b'}(\gamma_{b'})_i{}^j(\gamma^a)_{\alpha\beta} + \phi_{a_1a_2a_3}\delta_i^j(\gamma^{a_1a_2a_3})_{\alpha\beta}\,. \tag{4.13}$$

Equation (4.11) ensures the correct dynamics for the five brane [21, 25] and in particular implies that

$$D_{\alpha i}\Theta_{\beta}{}^{j} \sim -(\not{\partial})_{\alpha\beta}(\gamma_{n'})_i{}^jX^{n'} + \delta_i^j(\gamma_{n_1n_2n_3})_{\alpha\beta}h^{n_1n_2n_3} \tag{4.14}$$

where $h_{n_1n_2n_3}$ is the self-dual gauge field strength of the fivebrane. Hence , we find that the space-time derivatives of the scalar and second rank gauge fields are identified with the automorphism group [8]. The underlying consistency of equations (4.11) and (4.12) are ensured by existence of the (2,0) supermultiplet which they specify. We note that as B_{ab} has a gauge symmetry, the supersymmetry algebra will close only if one includes such transformations.

4.3 The four-dimensional Born-Infeld theory

The underlying supersymmetry algebra $\underline{K}$ for this model when written in two component notation is

$$\{Q_A, Q_B\} = 0 = \{S_A, S_B\},\ \{Q_A, Q_{\dot{B}}\} = -2i(\sigma^c)_{A\dot{B}} P_c = \{S_A, S_{\dot{B}}\}, \tag{4.15}$$

and

$$\{Q_A, S_{\dot{B}}\} = -2i(\sigma^c)_{A\dot{B}} Z_c\,. \tag{4.16}$$

The automorphism algebra contains the usual Lorentz rotations

$$[Q_A, J_{ab}] = \frac{1}{2}(\sigma_{ab})_A{}^B Q_B\,, \qquad [S_A, J_{ab}] = \frac{1}{2}(\sigma_{ab})_A{}^B S_B \tag{4.17}$$

and well as automorphisms with generators R_{ab} and R which obey the commutators

$$\begin{aligned} [Q_A, R_{ab}] &= \frac{1}{2}(\sigma_{ab})_A{}^B S_B\,, & [S_A, R_{ab}] &= \frac{1}{2}(\sigma_{ab})_A{}^B Q_B\,, \\ [Q_A, R] &= S_A\,, & [S_A, R] &= Q_A\,. \end{aligned} \tag{4.18}$$

The super Jacobi identities determine the remaining commutators to be

$$\begin{aligned} [R_{ab}, R_{cd}] &= J_{ad}\eta_{bc} + \cdots\,, & [R_{ab}, J_{cd}] &= R_{ad}\eta_{bc} + \cdots\,, \\ [R_{ab}, R] &= 0\,, & [J_{ab}, R] &= 0\,, \end{aligned} \tag{4.19}$$

and

$$\begin{aligned} [P_c, R_{ab}] &= -\eta_{ac} Z_b + \eta_{bc} Z_a\,, & [Z_c, R_{ab}] &= -\eta_{ac} P_b + \eta_{bc} P_a\,, \\ [P_c, R] &= 2Z_c\,, & [Z_c, R_{ab}] &= 2P_c \end{aligned} \tag{4.20}$$

where in these equations $+\cdots$ means one must add the corresponding terms required by anti-symmetry. We note that this algebra has an additional central charge Z_a compared to the usually used $N = 2$ supersymmetry algebra and this allows the presence of the additional automorphism R_{ab}. The generators R is just part of the usual SU(2)⊗U(1) R symmetry of $N = 2$ supersymmetry.

Leaving aside the generator R, the above algebra is just two copies of the standard $N = 1$ supersymmetry algebra. If we define

$$\psi_A^\pm = Q_A \pm S_A\,, \qquad P_a^\pm = P_a \pm Z_a\,, \qquad J_{ab}^\pm = \frac{1}{2}(J_{ab} \pm R_{ab}) \tag{4.21}$$

they obey

$$\begin{aligned} \{\psi_A^\pm, \psi_{\dot{B}}^\pm\} &= -2i(\sigma^c)_{A\dot{B}} P_c^\pm\,, & \{\psi_A^\pm, \psi_B^\pm\} &= 0\,, \\ [\psi_A^\pm, J_{ab}^\pm] &= \frac{1}{2}(\sigma_{ab})_A{}^B \psi_B^\pm\,, & [P_a^\pm, J_{bc}^\pm] &= -\eta_{ab} P_c^\pm + \eta_{ac} P_b^\pm\,. \end{aligned} \tag{4.22}$$

The two copies of the $N = 1$ supersymmetry algebras (anti-)commute with each other. The commutator with R is $[\psi^\pm, R] = \pm\psi^\pm$. Two separate algebras are mixed together in the dynamics as the preserved space-time translations, supercharges and Lorentz algebra are a superposition of generators from the two separate algebras.

Starting from the algebra of equation (4.15) and (4.16), but with Z_a absent and no automorphism generators as given in equation (4.18), except the Lorentz algebra of equation (4.17), the full four dimensional Born Infeld theory was constructed [3] as a non-linear realisation. These authors also suggested that the auxiliary field present in this model was related to the part of the usual R symmetry algebra.

We now consider the non-linear realisation of the algebra of equations (4.15), (4.16) and (4.17) with the local sub-algebra being just the Lorentz algebra with generators J_{ab}. In the notation of the beginning of this section the preserved supercharges are $Q_\alpha = (Q_A, Q^{\dot{A}})$ while the broken supercharges are $Q_{\alpha'} = (S_A, S^{\dot{A}})$. The corresponding group element takes the from

$$g = \exp(x^a P_a + \theta^A Q_A + \theta^{\dot{A}} Q_{\dot{A}}) \exp(W^A S_A + W^{\dot{A}} S_{\dot{A}}) \exp(A^a Z_a) \exp(\phi^{ab} R_{ab} + \phi R) \,. \quad (4.23)$$

The Cartan forms of equation (3.10) of the linearised theory become in this case

$$\nabla_A A^a = D_A A^a + 2i(\sigma^a)_{A\dot{B}} W^{\dot{B}} \,, \qquad \nabla_{\dot{A}} A^a = D_{\dot{A}} A^a + 2i(\bar{\sigma}^a)_{\dot{A}B} W^B \,, \quad (4.24)$$

While those of equation (3.11) become

$$\begin{aligned} \nabla_B W^A &= D_B W^A + \frac{1}{2}(\sigma^{ab})_B{}^A \phi_{ab} + \delta_A^B \phi \,, \qquad \nabla_{\dot{B}} W^A = D_{\dot{B}} W^A \,, \\ \nabla_{\dot{B}} W^{\dot{A}} &= D_{\dot{B}} W^{\dot{A}} - \frac{1}{2}(\sigma^{ab})_{\dot{B}}{}^{\dot{A}} \phi_{ab} + \delta_{\dot{A}}^{\dot{B}} \phi \,, \qquad \nabla_B W^{\dot{A}} = D_B W^{\dot{A}} \,, \end{aligned} \quad (4.25)$$

The Grassmann even constraints of equation (3.14) then imply that

$$D_{\dot{B}} W^A = 0 = D_B W^{\dot{A}} \,, \qquad \text{and} \qquad D_B W^B = D_{\dot{B}} W^{\dot{B}} \quad (4.26)$$

where we have taken into account of the reality of ϕ. We note that in this case the Grassmann even constraints have not only determined the automorphism Goldstone bosons of the theory, but they have also provided the conditions that specify the gauge covariant theory. Equations (4.25) and (4.26) are the correct linearised superspace constraints of the Born-Infeld theory off shell. In particular they imply that the field strength and auxiliary field of the theory are identified with the Goldstone bosons for the automorphism ϕ_{ab} and ϕ. The Grassmann odd constraints of equation (3.14), which in this case are those of (4.24), are the correct superspace constraints between the vector potential and Goldstino. Thus we find that the general scheme given above also works for this model. It would be interesting to complete the calculation to the full non-linear theory.

5. E_{11} formulation of brane dynamics

The M two brane was constructed long ago [13] by demanding that it possess κ-symmetry which can be viewed as part of world volume supersymmetry [12]. When coupled to the eleven dimensional supergravity background the two brane also possess local supersymmetry. However, it then only possess κ-symmetry, which is essential for its consistency, if the background supergravity fields satisfy their equations of motion [13]. Thus the two brane possess all the manifest symmetries of the background supergravity theory and it also implies the equation of motion of the background fields. This picture is also true for the other super branes in ten and eleven dimensions and in this sense they are more fundamental than the supergravity theories to which they couple. As such, if the supergravity theories can be extended to possess the Kac-Moody algebra E_{11} [9] one might think that this symmetry should also be present in a suitably extended formulation of brane dynamics. Another way of arriving at this idea is to recall that open string scattering leads to closed strings and at low energy the latter are described by the corresponding supergravity theories. If the supergravity theories when suitably extended possess an E_{11} symmetry then this should arise from the open strings and should be present in their dynamics.

In fact, there is a connection between brane charges and E_{11}. There is considerable evidence [11, 14, 16] that the brane charges to belong to the fundamental representation, denoted l_1, of algebra E_{11} associated with the node at the end of the longest tail of the E_{11} Dynkin diagram. The lowest level such charges are given by [11]

$$P_{\underline{a}}\,; \quad Z^{\underline{a}\underline{b}}\,; \quad Z^{\underline{a}_1\cdots\underline{a}_5}\,; \quad Z^{\underline{a}_1\cdots\underline{a}_7,\underline{a}_8}\,, \quad Z^{\underline{a}_1\cdots\underline{a}_8}\,; \quad Z^{\underline{b}_1\underline{b}_2\underline{b}_3,\underline{a}_1\cdots\underline{a}_8}\,; \quad \ldots \tag{5.1}$$

The first entry is the space-time translations and the next two can be identified with the central charges of the eleven dimensional supersymmetry algebra [11]. These three quantities are known [17] to be the brane charges of the point particle, two brane and five brane of M theory. The higher level objects in the l_1 representation should also correspond to brane-like objects in the extended theory which possess E_{11} symmetry. Indeed, it has been shown that for every element in the l_1 representation there is a corresponding field in the adjoint representation of E_{11} that belongs to the correct SL(11) representation to allow it to be coupled to a brane with the corresponding charge [11, 16]. There has been much discussion of the brane charges that result when the IIA string theory is dimensionally reduced on a torus [18, 19] and, in particular, it has been realised [18, 19] that the central charges that occur in the reduced supersymmetry algebra can not form multiplets of the U-duality algebras. However, decomposing the l_1 representation of the algebra appropriate to dimensional reduction one finds [11, 14] that it contains all the usual brane charges as well as more exotic objects that complete the U-duality multiplets. While it is inherent in the construction that the brane charges belong to U duality multiplets it is encouraging that all the expected brane charges belong to a single E_{11} representation and in this way one finds an eleven dimensional origin for the exotic charges required by U duality.

Since taking just the fields of the non-linear realisation of E_{11} to depend on just the usual coordinates of space-time, which are associated with $P_{\underline{a}}$, breaks E_{11}, it was proposed [11] that these fields should depend on an infinite set of coordinates which transform under E_{11} as the l_1 representation, that is the set of coordinates

$$X^{\underline{a}}\,, \quad X_{\underline{a}\underline{b}}\,; \quad X_{\underline{a}_1\ldots\underline{a}_5}\,; \quad X_{\underline{a}_1\ldots\underline{a}_7,\underline{a}_8}\,, \quad X_{\underline{a}_1\ldots\underline{a}_8}\,; \quad X_{\underline{b}_1\underline{b}_2\underline{b}_3,\underline{a}_1\ldots\underline{a}_8,}\,; \quad \ldots \tag{5.2}$$

From this perspective one may suppose that the dynamics of branes is E_{11} invariant and that it is constructed from fields which transform in the l_1 representation as in equation (5.2). Bosonic branes and superbranes can be viewed as defects in Minkowski space

JHEP03(2005)077

and superspace respectively and it is a natural generalisation to suppose that the bosonic sector of superbranes corresponds to a defect in a space with coordinates given in equation (5.2). The dynamical fields of the brane being Goldstone bosons for the brane charges which are spontaneously broken.

The result found earlier in this paper, namely that the world volume gauge fields and the usual transverse scalar fields of the brane have a common origin in that they are Goldstone bosons for the corresponding central charges provided encouragement for the idea of an underlying E_{11} symmetry of brane dynamics. Indeed, it implies that these fields must occur in the group element of the non-linear realisation of equation (3.7) in precisely the required way to be suitable to be embedded in an E_{11} formulation.

The question of whether the hidden symmetries of the supergravity theories also occurred in dimensional reduction of brane dynamics was given a partial answer in reference [20]. It was shown that a two brane reduced on a four torus did have the expected SL(5,R) symmetry. One interesting aspect of this work was that it used a formulation of the two brane which possessed an apparently adhoc field with two anti-symmetric indices as well as the usual scalar fields. However, it was noted [20] that this formulation could not account for the expected symmetries if the dimensional reduction was on a torus of dimension greater than four.

Clearly, a dimensional reduction of the brane dynamics proposed in this section on a torus would possess the same symmetries as the eleven dimensional supergravity theory under the same reduction. The decomposition of the l_1 representation appropriate to a dimensional reduction to three dimension was given in [14] and it was shown how the fields of equation (5.2) formed multiplets of the corresponding E_8 symmetry group. The results for a dimensional reduction on a smaller dimension torus can also be readily deduced from these. For a torus of dimension four it is easy to see that X^i, X_{ij} belong to the $\underline{4}$ and $\underline{6}$ of SL(4) and so form the $\underline{10}$ of SL(5), while for a torus of dimension five the $X^i, X_{ij}, X_{i_1\ldots i_5}$ transform as the $\underline{5}$, $\underline{10}$ and $\underline{1}$ of SL(5) and so form the $\underline{16}$ of SO(5,5). Here the indices $i, j, \ldots$ are in the directions in which the torus lies. However, for a seven torus the coordinates corresponding to central charges of the supersymmetry algebra will no longer suffice and one finds that $X^i, X_{ij}, X_{i_1\ldots i_5}, X_{i_1\ldots i_7,i}$, transform as the $\underline{7}$ and $\underline{21}$, $\underline{21}$ and $\underline{7}$ of SL(7) to form the $\underline{56}$ of E_7. On an eight torus one needs coordinates up to level six to form the $\underline{248}$ of E_8.

The usual formulation of the two brane involves only a $X^{\underline{a}}$ and so, at first sight, it does not appear to possess an E_{11} symmetry. However, we will now sketch an E_{11} formulation of brane dynamics and apply it to the two brane. We will find that at low levels it does indeed lead to the expected dynamics.

It will prove instructive to first recall the dynamics of a bosonic p brane in a flat background is given by the non-linear realisation of ISO(1,D-1)/SO(1,p)⊗SO(D-p-1) [8]. The group element is of the form

$$\exp(X^{\underline{a}} P_{\underline{a}}) \exp(\phi_a{}^{b'} J^a{}_{b'}) \tag{5.3}$$

where the fields depend on the world volume coordinates ξ^n. The Cartan form corresponding to the translations is given by $\nabla_n X^{\underline{a}} = \partial_n X^{\underline{p}} \Phi_{\underline{p}}{}^{\underline{a}}$, where $\Phi_{\underline{p}}{}^{\underline{a}} = (e^{\phi})_{\underline{p}}{}^{\underline{a}}$. This transforms only under SO(1,p)⊗SO(D-p-1) and as such we may set $\nabla_n X^{a'} = 0$ which implies that the space-time derivatives of $X^{n'}$ are given in terms of $\phi_a{}^{b'}$. The only covariant derivative of $X^{\underline{a}}$ remaining is $\nabla_n X^a \equiv f_n{}^a$ and the action is given by $\int d^{p+1}\xi \;\det f_n{}^a$. Using the identity $f_n{}^a \eta_{ab} f_n{}^b = \nabla_n X^{\underline{a}} \eta_{\underline{ab}} \nabla_m X^{\underline{b}} = \partial_n X^{\underline{p}} \eta_{\underline{pq}} \partial_m X^{\underline{q}} \equiv \gamma_{nm}$ we recognise the familiar expression for the action.

We may generalise the above to include the coupling of the bosonic brane to the gravity background [10] by taking as our group element

$$\exp(X^{\underline{a}} P_{\underline{a}}) \exp(h_{\underline{a}}{}^{\underline{b}} K^{\underline{a}}{}_{\underline{b}}) \tag{5.4}$$

where $K^{\underline{a}}{}_{\underline{b}}$ are the generators of GL(D) and we can identify $e_{\underline{n}}{}^{\underline{a}} = (\exp h)_{\underline{n}}{}^{\underline{a}}$ as the vierbein from the gravity that results from the group element [10]. The corresponding covariant derivative of $X^{\underline{a}}$ is given by $\nabla_n X^{\underline{a}} = \partial_n X^{\underline{p}} e_{\underline{p}}{}^{\underline{a}}$. We define $\nabla_n X^{\underline{a}} \eta_{\underline{ab}} \nabla_m X^{\underline{b}} = \partial_n X^{\underline{p}} g_{\underline{pq}} \partial_m X^{\underline{q}} \equiv \gamma_{nm}$. The action is then given by $\int d^{p+1}\xi \sqrt{-\det \gamma_{nm}}$. The anti-symmetric part of $h_a{}^{b'}$ play the role of Goldstone boson for the broken Lorentz symmetry.

We now sketch the E_{11} non-linear realisation of a super brane dynamics in the supergravity background. We will consider only the bosonic fields and work to second order in the brane coordinates and the background supergravity fields. Hence we consider the group element built from the algebra, denoted $E_{11} \otimes_s l_1$, which is the semi-direct product of E_{11} with generators that transform in the l_1 representation of E_{11}. The group element is given by

$$\begin{aligned} g &= \exp(X^{\underline{a}} P_{\underline{a}}) \exp(X_{\underline{ab}} Z^{\underline{ab}}) \exp(X_{\underline{a}_1 \cdots \underline{a}_5} Z^{\underline{a}_1 \cdots \underline{a}_5}) \times \cdots \\ &\quad \times \exp(h_{\underline{a}}{}^{\underline{b}} K^{\underline{a}}{}_{\underline{b}}) \exp\left(\frac{1}{3!} A_{\underline{a}_1 \underline{a}_2 \underline{a}_3} R^{\underline{a}_1 \underline{a}_2 \underline{a}_3}\right) \exp\left(\frac{1}{6!} A_{\underline{a}_1 \cdots \underline{a}_6} R^{\underline{a}_1 \cdots \underline{a}_6}\right) \cdots \\ &\equiv g_1 \exp(h_{\underline{a}}{}^{\underline{b}} K^{\underline{a}}{}_{\underline{b}}) \exp\left(\frac{1}{3!} A_{\underline{a}_1 \underline{a}_2 \underline{a}_3} R^{\underline{a}_1 \underline{a}_2 \underline{a}_3}\right) \exp\left(\frac{1}{6!} A_{\underline{a}_1 \cdots \underline{a}_6} R^{\underline{a}_1 \cdots \underline{a}_6}\right) \cdots \\ &\equiv g_1 g_2 \,. \end{aligned} \tag{5.5}$$

The fields associated with l_1 depend on the coordinates ξ^n which parameterise the brane world volume and those of the supergravity background depend on the fields in l_1. The commutators of these generators can be found in [9, 11]. We will in this sketch take as the local sub-algebra only the Lorentz algebra.

The Cartan forms are given by

$$g^{-1} dg = \nabla_n X^{\underline{a}} P_{\underline{a}} + \nabla_n X_{\underline{ab}} Z^{\underline{ab}} + \nabla_n X_{\underline{a} \cdots \underline{a}_5} Z^{\underline{a} \cdots \underline{a}_5} + \cdots + g_2^{-1} dg_2 \,. \tag{5.6}$$

The latter are the Cartan forms associated with E_{11} algebra. We find that

$$\begin{aligned} \nabla_n X^{\underline{a}} &= \partial_n X^{\underline{p}} e_{\underline{p}}{}^{\underline{a}} \,, \qquad \nabla_n X_{\underline{ab}} = \partial_n X_{\underline{pq}} (e^{-1})_{\underline{a}}{}^{\underline{p}} (e^{-1})_{\underline{b}}{}^{\underline{q}} - \partial_n X^{\underline{p}} e_{\underline{p}}{}^{\underline{c}} A_{\underline{cab}} \\ \nabla_n X_{\underline{a}_1 \cdots \underline{a}_5} &= \partial_n X_{\underline{p}_1 \cdots \underline{p}_5} (e^{-1})_{\underline{a}_1}{}^{\underline{p}_1} \cdots (e^{-1})_{\underline{a}_5}{}^{\underline{p}_5} + \\ &\quad + \partial_n X^{\underline{q}} e_{\underline{q}}{}^{\underline{c}} (A_{\underline{c}\underline{a}_1 \cdots \underline{a}_5} + 10 A_{\underline{c}[\underline{a}_1 \underline{a}_2} A_{\underline{a}_3 \cdots \underline{a}_5]}) - \\ &\quad - 20 A_{[\underline{a}_1 \underline{a}_2 \underline{a}_3} \partial_n X_{\underline{n}_4 \underline{n}_5} (e^{-1})_{\underline{a}_4}{}^{\underline{n}_4} (e^{-1})_{\underline{a}_5]}{}^{\underline{n}_5} \,. \end{aligned} \tag{5.7}$$

JHEP03(2005)077

We now apply this to the two brane taking field only to next to lowest order. The equation which is first order in derivatives, constructed from the Cartan form and world volume reparmetisation invariant is given by

$$E^n_{\underline{a}} \equiv \sqrt{-\gamma}\gamma^{nm}\nabla_n X^{\underline{b}}\eta_{\underline{ab}} + d_1\epsilon^{nmr}\nabla_m X_{\underline{ab}}\nabla_r X^{\underline{b}} + \cdots = 0 \tag{5.8}$$

where

$$\gamma_{nm} \equiv \nabla_n X^{\underline{a}}\nabla_n \eta_{\underline{ab}} X^{\underline{b}} + \cdots \tag{5.9}$$

and d_1 is a constant. Using the above expressions for the Cartan forms this equation become

$$e_{\underline{n}}{}^{\underline{a}}E^n_{\underline{a}} = \sqrt{-\gamma}\gamma^{nm}\partial_n X^{\underline{p}}g_{\underline{pn}} + d_1\epsilon^{nmr}\partial_m X_{\underline{np}}\partial_r X^{\underline{p}} - d_1\epsilon^{nmr}\partial_m X^{\underline{q}}A_{\underline{qnp}}\partial_r X^{\underline{p}} + \cdots = 0\,. \tag{5.10}$$

Taking the derivative ∂_n of this equation we find at lowest order the correct equation of motion for the two brane in a constant background fields. For a non-constant background we would have an equation which is second order in derivatives. We note that this equation contains an additional field $X_{\underline{np}}$ which is dynamical in the sense that it comes with derivatives, but it does not lead to more degrees of freedom as the equation of motion is first order. Thus it is consistent with the considerations discussed earlier in this paper. We note that the inclusion of fermions would require more than just the Goldstino of section 4.

By wrapping the two brane in eleven dimensions on a circle one may take the above dynamics and obtain the analogous results for the dynamcis of the IIA string in ten dimensions. The lowest level coordinates are $X^{\underline{a}}$, $a = 0, 1, \ldots 9$ and $\bar{X}_{\underline{a}} \equiv X_{\underline{a}11}$. Making the decomposition of the l_1 representation into D_{10} representations by deleting the last node on the gravity line of the Dynkin diagram one finds [14] that the lowest level multiplet contains the $X^{\underline{a}}$ and $\bar{X}_{\underline{a}}$ which belong to the vector representation of D_{10}. Hence, just keeping these fields one will find a D_{10} invariant dynamics which was discussed in reference [26]. In this case, the additional coordinate $\bar{X}_{\underline{a}}$ encodes the possibility of interchanging Kaluza-Klein and winding modes.

We now also sketch the five brane dynamics from this viewpoint. In this case we must include the fields at the next level. The analogous equation for the scalar fields is

$$\gamma^{nm}\nabla_n X^{\underline{b}}\eta_{\underline{ab}} + d_2\epsilon^{nm_1\ldots m_5}\nabla_{m_1}X_{\underline{ba}_1\ldots\underline{a}_5}\nabla_{m_2}X^{\underline{a}_1}\cdots\nabla_{m_5}X^{\underline{a}_4} + \cdots = 0 \tag{5.11}$$

where d_2 is a constant. While for the two form $X^{\underline{ab}}$ the equation is

$$\nabla_{[n}X_{\underline{ab}}\nabla_m X^{\underline{a}}\nabla_{p]}X^{\underline{b}} + \cdots \quad \text{is self dual}\,. \tag{5.12}$$

Substituting for the Cartan forms one recognises the correct equations of motion for the five brane [25] in a constant background up to the level considered. Since any p brane has a coordinate in the l_1 representation that has p anti-symmetrised indices we recognise that equations (5.8) and (5.11) are just special cases of an obvious generalisation that gives the equation of motion of the embedding coordinates.

We will give a more systematic account of the low level dynamics of branes viewed as a non-linear realisation of E_{11} elsewhere. In particular, we will address the issue, glossed over here, of what are the correct local sub-algebras which will in turn determine the field content of the non-linear realisations. For the five brane we expect to extend the local sub-algebra to include the Lorentz algebra and the automorphism discussed in equation (4.9) and we expect that this will lead to the peculiar factors of the three form field strength that occur in the five brane dynamics. The brane dynamics in the absence of background fields is likely to be a non-linear realisation of the l_1 representation together with the Cartan involution invariant sub-algebra of E_{11}, or possibly a sub-algebra of this.

Despite the low level approximation of the above dynamics the correct equations of motion appear naturally and, to those with a sympathetic disposition, it may encourage the belief in an underlying E_{11} symmetry of brane dynamics.

6. Discussion

Inspired by Matrix Theory [24], it has been argued [18] that the maximal $d+1$ dimensional "super Yang-Mills theory" dimensional reduced on a d-dimensional torus has an E_d, $d \leq 9$ symmetry where E_d is the hidden symmetry that appears when eleven dimensional supergravity is reduced on a suitable dual torus of dimension d. In particular, it was found that this hidden symmetry, or at lest the relevant discrete part of it, is generated in the "super Yang-Mills theory" by simply permuting the radii of compactification combined with Montonen-Olive duality. The authors of [18] raised the question of what extension of super Yang-Mills theory could possess such a symmetry.

In this paper we have argued that brane dynamics, like the supergravity theories to which it couples, can be extended to admit an E_{11} symmetry. Indeed, both are non-linear realisation of E_{11}; the supergravity theory being the low level approximation involving the adjoint representation while the usual brane dynamics is the low level approximation from a non-linear realisation which also includes the l_1 representation. It is important to note that the hidden symmetries are actually part of the theory before dimensional reduction and so appear automatically when they are dimensionally reduced and from this perspective it is inevitable that the supergravity and brane dynamics, when suitably extended, have the same symmetries when dimensionally reduced. This is in contrast to the mismatch between the brane and M theory symmetries noted in [18]. The approach advocated here also answers the another question raised in [18], namely it provided a higher dimensional origin for the exotic charges whose existence in the Yang-Mills theory is implied by acting with the analogue of the U-duality transformations on the well known charges.

The ideas put forward in this paper are very natural from the viewpoint of the gauge-gravity correspondences (the Maldacena conjecture) since the two theories could be viewed as just different faces of their common underlying E_{11} symmetry. We would note that many of the checks of the Maldacena conjecture are really a consequence of the symmetries of the gauge and gravity theories. The correspondence between the supergravity and brane dynamics implied by their common symmetries could be exploited to give a mapping between quantities in the two theories. It would be interesting to see if these were the same as those that appear in Matrix theory [24].

JHEP03(2005)077

The l_1 representation contains at level zero the usual space-time coordinates $X^{\underline{a}}$. Using techniques similar to those used in reference [16], it is straightforward to show, modulo some very unexpected conspiracy discussed in that reference, that the l_1 representation also contains elements with the same Sl(10) representation content as the space-time coordinates at levels $n_c = 11m, \ m \in \mathbf{Z}$. Thus it contains multiple copies of the space-time like coordinates first found at level zero. Indeed, one can show that if a representation of Sl(10) occurs at level n_c it also occurs at level $n_c + 11$. As a result, one finds in effect multiple copies of the first few coordinates used to describe brane dynamics. There is one intriguing interpretation of this repetition; it might be the E_{11} way of encoding non-abelian branes. One might interpreted the first set of coordinates as belonging to a single brane and the subsequent sets as belonging to multiple branes. One test of this idea would be to see if the multiplicities of the higher coordinates are consistent with this picture in that they make up an U(m) adjoint multiplet. While one might expect the generators P_a at level zero to commute it could well be the case that the higher level generators with the same indices do not commute.

Acknowledgments

I wish to thank Jonathan Bagger and David Olive for useful discussions. This research was supported by a PPARC senior fellowship PPA/Y/S/2002/001/44 and in part by the PPARC grants PPA/G/O/2000/00451 and the EU Marie Curie, research training network grant HPRN-CT-2000-00122.

References

[1] S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological lagrangians, 1*, *Phys. Rev.* **177** (1969) 2239;
D. Volkov, *Phenomenological lagrangians*, *Sov. J. Part. Nucl.* **4** (1973) 3.

[2] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, *Phys. Lett.* **B 180** (1986) 370;
J. Hughes and J. Polchinski, *Partially broken global supersymmetry and the superstring*, *Nucl. Phys.* **B 278** (1986) 147.

[3] J. Bagger and A. Galperin, *A new goldstone multiplet for partially broken supersymmetry*, *Phys. Rev.* **D 55** (1997) 1091 [hep-th/9608177].

[4] J. Bagger and A. Galperin, *Matter couplings in partially broken extended supersymmetry*, *Phys. Lett.* **B 336** (1994) 25 [hep-th/9406217].

[5] E. Ivanov and S. Krivonos, $N = 1$ $D = 4$ *supermembrane in the coset approach*, *Phys. Lett.* **B 453** (1999) 237 [hep-th/9901003].

[6] J. Bagger and A. Galperin, *The tensor goldstone multiplet for partially broken supersymmetry*, *Phys. Lett.* **B 412** (1997) 296 [hep-th/9707061].

[7] D.V. Volkov and V.P. Akulov, *Is the neutrino a goldstone particle?*, *Phys. Lett.* **B 46** (1973) 109.

[8] P.C. West, *Automorphisms, non-linear realizations and branes*, *JHEP* **02** (2000) 024 [hep-th/0001216].

JHEP03(2005)077

[9] P.C. West, *E(11) and M-theory*, *Class. and Quant. Grav.* **18** (2001) 4443 [hep-th/0104081].

[10] P.C. West, *Hidden superconformal symmetry in M-theory*, *JHEP* **08** (2000) 007 [hep-th/0005270].

[11] P. West, $E_1 1$*,* Sl$_3$2 *and central charges*, *Phys. Lett.* **B 575** (2003) 333 [hep-th/0307098].

[12] D. Sorokin, V. Tkach and D.V. Volkov, *Superparticles, twistors and Siegel symmetry*, *Mod. Phys. Lett.* **A 4** (1989) 901;
D. Sorokin, V. Tkach, D.V. Volkov and A. Zheltukhin, *From superparticle Siegel supersymmetry to the spinning particle proper-time supersymmetry*, *Phys. Lett.* **B 259** (1989) 302.

[13] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Properties of the eleven-dimensional super membrane theory*, *Ann. Phys. (NY)* **185** (1988) 330; *Supermembranes and eleven-dimensional supergravity*, *Phys. Lett.* **B 189** (1987) 75.

[14] P. West, E_{11} *origin of brane charges and u-duality multiplets*, *JHEP* **08** (2004) 052 [hep-th/0406150].

[15] O. Barwald and P.C. West, *Brane rotating symmetries and the fivebrane equations of motion*, *Phys. Lett.* **B 476** (2000) 157 [hep-th/9912226].

[16] A. Kleinschmidt and P. West, *Representations of* $\mathcal{G}^{+++}$ *and the role of space-time*, *JHEP* **02** (2004) 033 [hep-th/0312247].

[17] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, *Topological extensions of the supersymmetry algebra for extended objects*, *Phys. Rev. Lett.* **63** (1989) 2443.

[18] S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, *Algebraic aspects of matrix theory on* T^d, *Nucl. Phys.* **B 509** (1998) 122 [hep-th/9707217];
N.A. Obers, B. Pioline and E. Rabinovici, *M-theory and u-duality on* T^d *with gauge backgrounds*, *Nucl. Phys.* **B 525** (1998) 163 [hep-th/9712084];
N.A. Obers and B. Pioline, *U-duality and M-theory, an algebraic approach*, hep-th/9812139; *U-duality and M-theory*, *Phys. Rept.* **318** (1999) 113 [hep-th/9809039].

[19] B. de Wit and H. Nicolai, *Hidden symmetries, central charges and all that*, *Class. and Quant. Grav.* **18** (2001) 3095 [hep-th/0011239].

[20] M.J. Duff and J.X. Lu, *Duality rotations in membrane theory*, *Nucl. Phys.* **B 347** (1990) 394.

[21] P.S. Howe and E. Sezgin, $D = 11$*,* $p = 5$, *Phys. Lett.* **B 394** (1997) 62 [hep-th/9611008].

[22] J.P. Gauntlett, K. Itoh and P.K. Townsend, *Superparticle with extrinsic curvature*, *Phys. Lett.* **B 238** (1990) 65.

[23] E. Ivanov and V. Ogievetsky, *The inverse Higgs mechanism in non-linear realisations*, *Teor. Mat. Fiz.* **25** (1975) 164.

[24] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[25] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008].

[26] S. Cecotti, S. Ferrara and L. Girardello, *Hidden noncompact symmetries in string theory*, *Nucl. Phys.* **B 308** (1988) 436;
M.J. Duff, *Duality rotations in string theory*, *Nucl. Phys.* **B 335** (1990) 610.